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Publication date:
2013

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Citation (APA):

Marmaras, K. (Author), Stolpe, M. (Author), Lund, E. (Author), & Sørensen, R. (Author). (2013). Optimal Design of Composite Structures by Advanced Mixed Integer Nonlinear Optimization. Sound/Visual production (digital)

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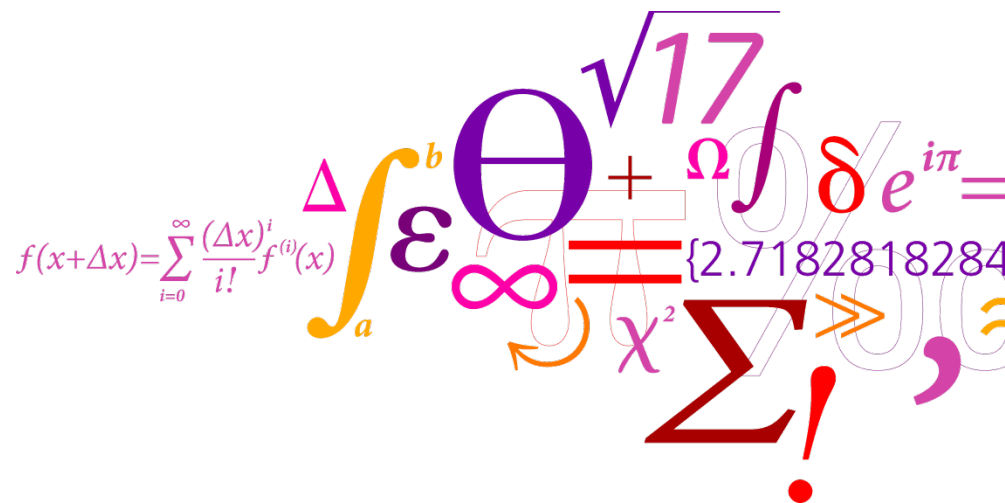
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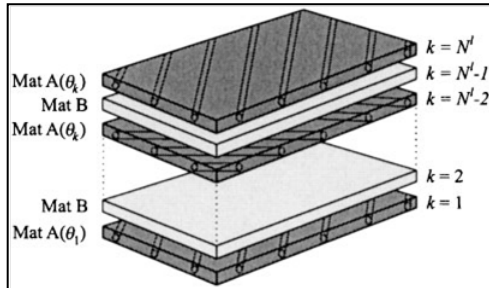
Optimal Design of Composite Structures by Advanced Mixed Integer Nonlinear Optimization

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- Professor Erik Lund – AAU
- PhD Student René Sørensen - AAU



Composite Materials

□ Composite Material



=

□ Fibres

- Glass Fibres
- Carbon Fibres
- Cellulose Fibres
- and more ...



Glass fibres

□ Matrix

- Polymers
- Metals
- Ceramics



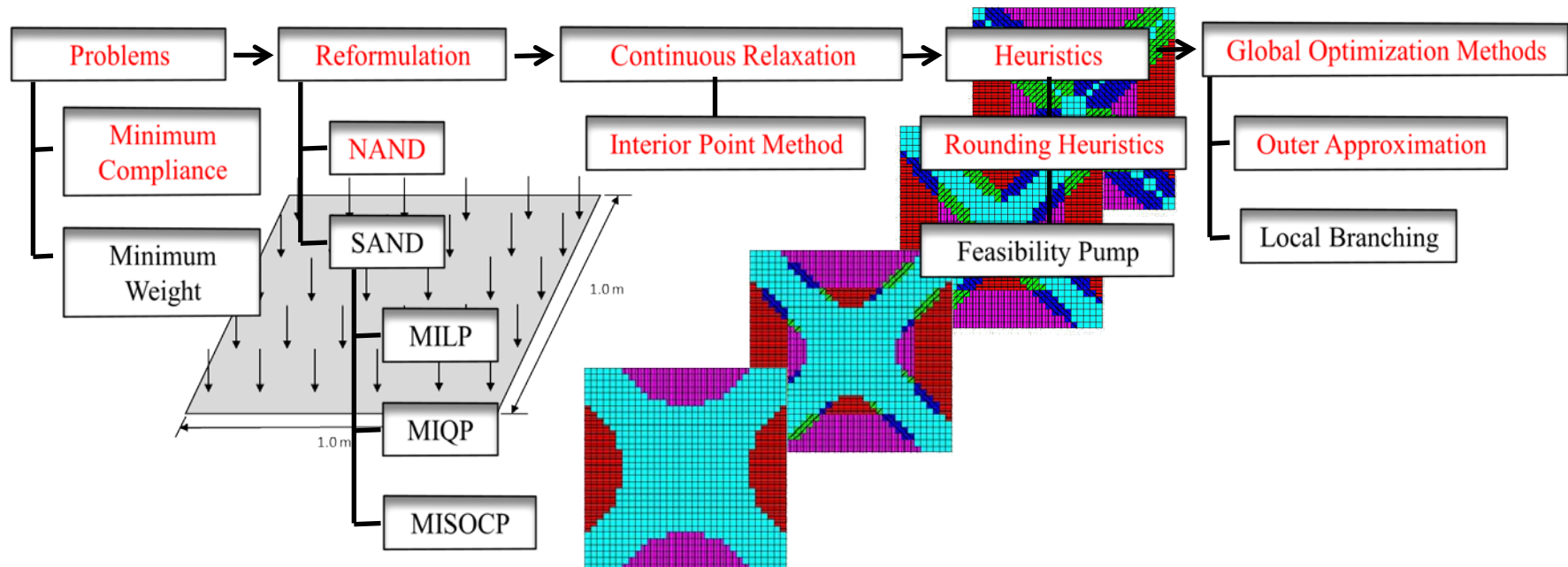
Polyester matrix



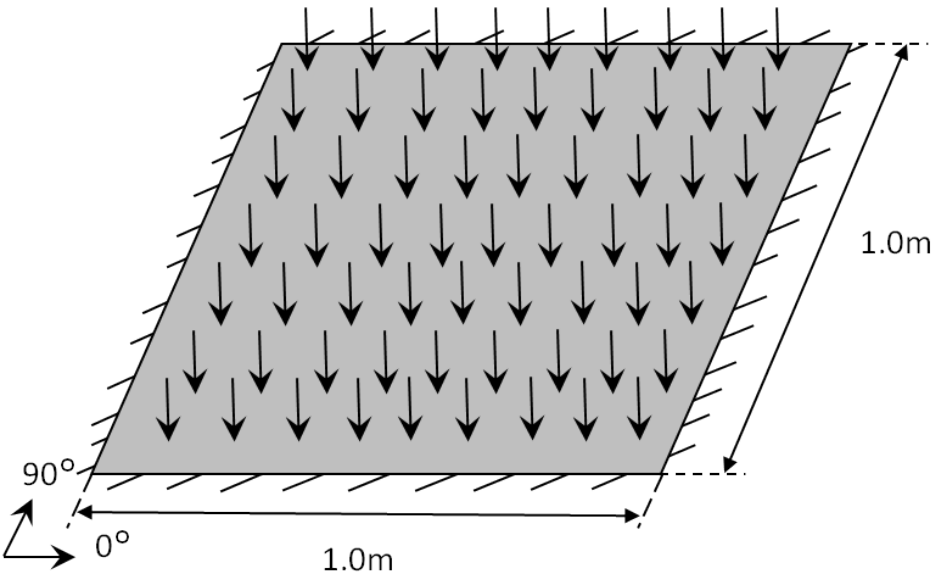
Optimal Design of Laminated Composite Structures

□ Design Parameterization

- Number of Layers
- **Fiber Orientation** (the considered case)
- Layer Thickness



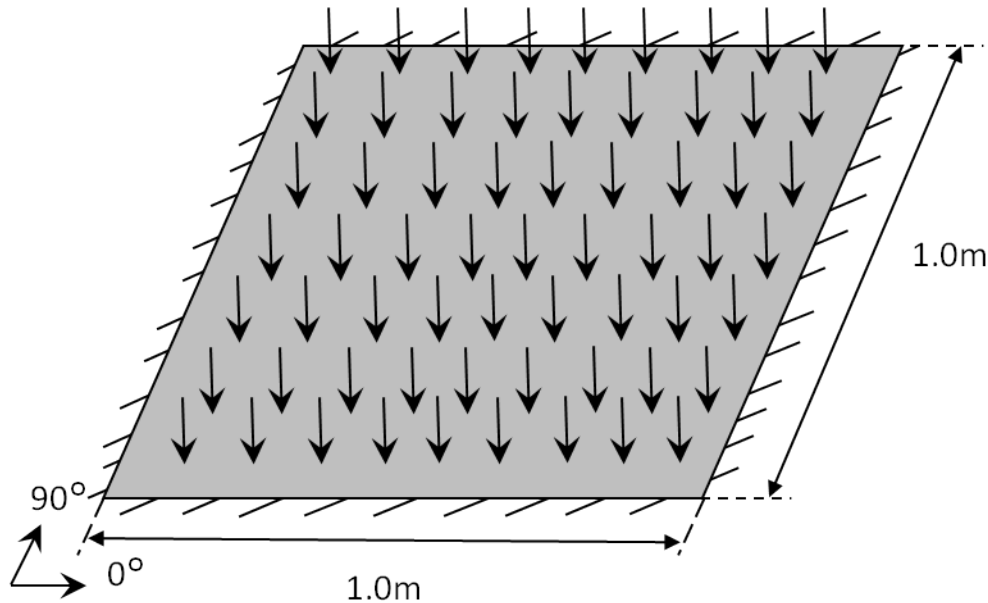
Numerical Example – Multi Layered Clamped Plate



- Distributed Out Of Plane (Static) Loading
- Clamped plate 1.0m x 1.0m x 0.005m
- 8 layers of equal thickness
- 5 candidate materials
 - Orthotropic - Glass fiber reinforced epoxy
 $\{ -45^\circ, 0^\circ, 45^\circ, 90^\circ \}$
 - Isotropic polymeric foam (35% of the domain)

	Foam	Glass Epoxy
E_x [Pa]	$1.25 \cdot 10^8$	$5.4 \cdot 10^{10}$
$E_y=E_z$ [Pa]		$1.8 \cdot 10^{10}$
$G_{xy}=G_{yz}=G_{xz}$ [Pa]		$0.9 \cdot 10^{10}$
Poisson's ratio	0.3	0.25
Density [kg/m ³]	100	1900

Numerical Example – Multi Layered Clamped Plate



- All materials behave linearly elastic
- Finite Element Formulations
 - Equivalent Single Layer Theory
 - First Order Shear Deformation Theory
 - Q9 plate/shell elements

Minimum Compliance (Maximum Stiffness)

minimize Compliance

subject to Equilibrium equations
Weight constraint
Single material selection per layer and element (DMO)



$$\begin{aligned} &\underset{x,u}{\text{minimize}} && \sum_{l=1}^L w_l f_l^T u_l \\ &\text{subject to} && \frac{K(x)u_l - f_l}{m(x)} = 0, \quad l = 1, \dots, L \\ & && m(x) \leq m_c, \\ & && \sum_{i=1}^n x_{ijk} = 1, \quad \forall (j, k) \\ & && x_{ijk} \in \{0, 1\}, \quad \forall (i, j, k) \end{aligned}$$

❑ Difficult combinatorial problem.

❑ The stiffness matrix is

- linear in the design variables. $K(x) = \sum_{ijk} x_{ijk} K_{ijk} = \sum_{ijk} x_{ijk} B_j^T C_{ik} B_j = \sum_j B_j^T (\sum_{ik} x_{ijk} C_{ik}) B_j$
- symmetric and positive definite.

❑ The nodal displacements are eliminated from the problem formulation.

$$u_l = K(x)^{-1} f_l$$

Reformulation - Nested Analysis and Design

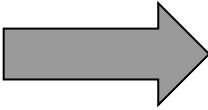

- The original non-convex 0-1 problem is reformulated as a 0-1 program with a convex objective function.

minimize	Compliance		$\underset{x}{\text{minimize}} \quad c(x) = \sum_{l=1}^L \frac{w_l f_l^T K(x)^{-1} f_l}{}$	←
subject to	Weight constraint Single material selection per layer and element (DMO)	→	$\text{subject to} \quad m(x) \leq m_c,$ $\sum_{i=1}^n x_{ijk} = 1, \quad \forall (j, k)$ $x_{ijk} \in \{0, 1\}, \quad \forall (i, j, k)$	

- The reformulated 0-1 program includes only the binary design variables.
- The equilibrium equations are now solved as part of computing the objective function.

Continuous Relaxation - Interior Point Method

□ By relaxing the integer constraints on the design variables we get the continuous relaxation of the minimum compliance problem.

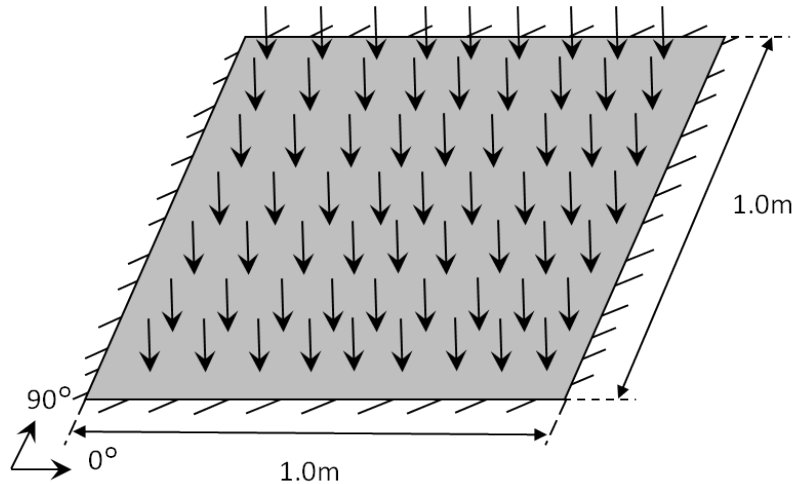
minimize	Compliance		minimize _{x}	$c(x) = \sum_{l=1}^L w_l f_l^T K(x)^{-1} f_l$	
subject to	Weight constraint		subject to	$m(x) \leq m_c$	
	Single material selection per layer and element (DMO)			$\sum_{i=1}^n x_{ijk} = 1,$	$\forall(j, k)$
				$\underline{x_{ijk} \geq 0},$	$\forall(i, j, k)$

□ The continuous problem has a non empty feasible set.

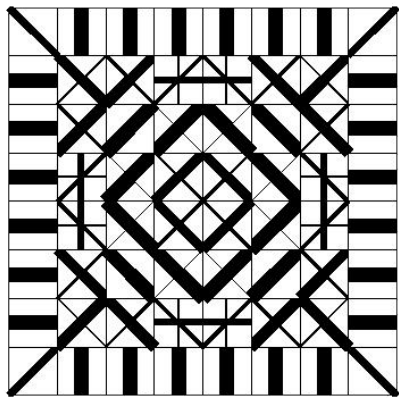
□ There is at least one optimal solution of the continuous problem.

□ The continuous relaxation of the mixed integer problem is solved by a primal-dual interior point method.

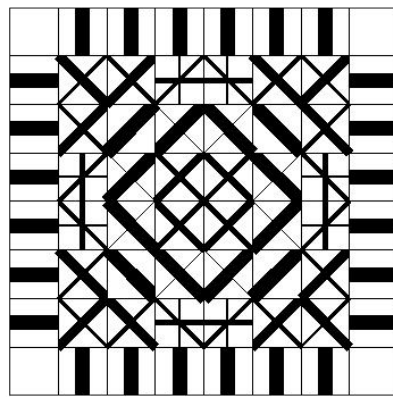
Numerical Example – Multi Layered Clamped Plate Continuous Relaxation



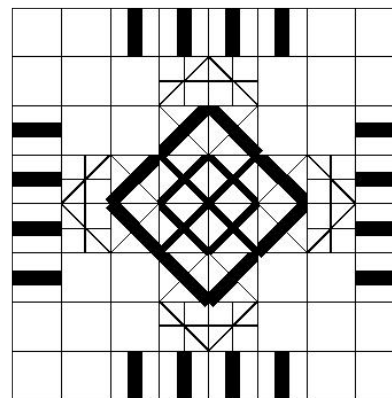
- Material distribution of the top four layers (due to symmetry) for a mesh discretization of 64 elements.
- The primal-dual interior point method converged in 18 iterations.
- The primal-dual interior point method has the ability to react swiftly to changes of scale of the problem.



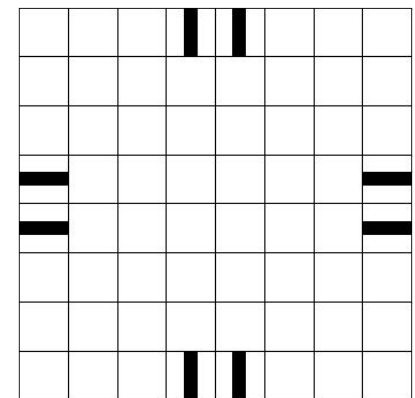
Layer 1



Layer 2



Layer 3



Layer 4

Heuristic

minimize Distance to 0-1 design

subject to Weight constraint

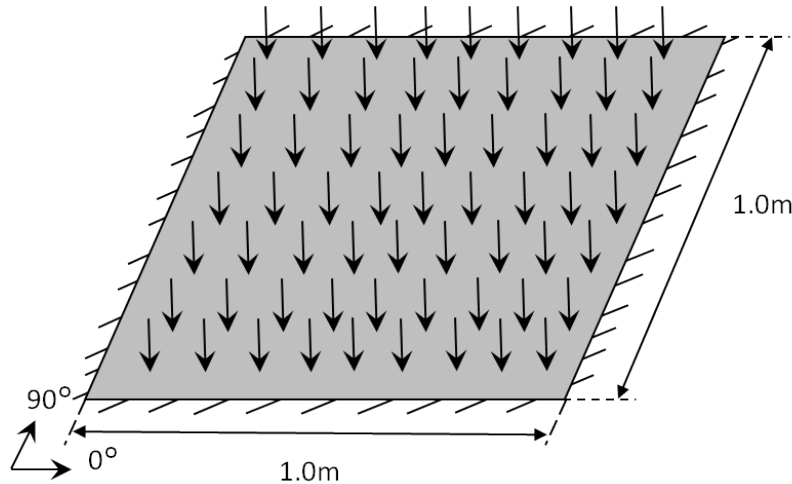
Single material
selection per layer
and element
(DMO)



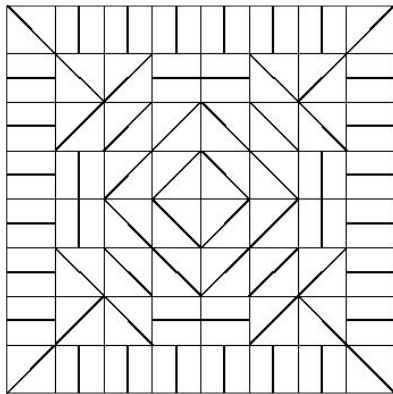
$$\begin{aligned}
 &\underset{x}{\text{minimize}} && \frac{\|x^* - x\|_1}{m(x)} \quad \leftarrow \\
 &\text{subject to} && m(x) \leq m_c \\
 & && \sum_{i=1}^{N^i} x_{ijk} = 1, \quad \forall(j, k) \\
 & && x_{ijk} \in \{0, 1\}, \quad \forall(i, j, k)
 \end{aligned}$$

- ☐ Rounds the optimal solution of the continuous relaxation .
- ☐ The method is guaranteed to find a feasible design.
- ☐ There are no guarantees on the quality of the obtained design.

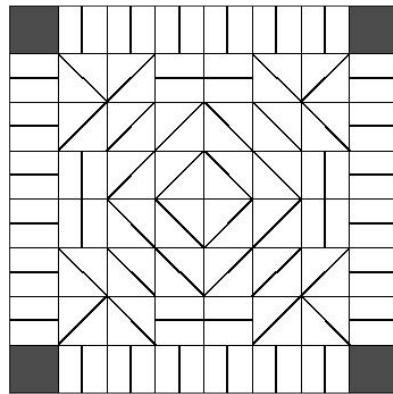
Numerical Example – Multi Layered Clamped Plate Rounding Heuristic



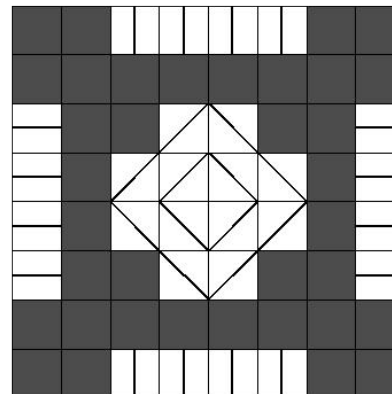
- Material distribution of the top four layers (due to symmetry) for a mesh discretization of 64 elements and a relative optimality gap of 1.23%.
- The optimized designs use the soft material to form a sandwich structure, leading to a high bending stiffness to weight ratio for the composite.
- The fibre angles are oriented according to the principal stress directions.



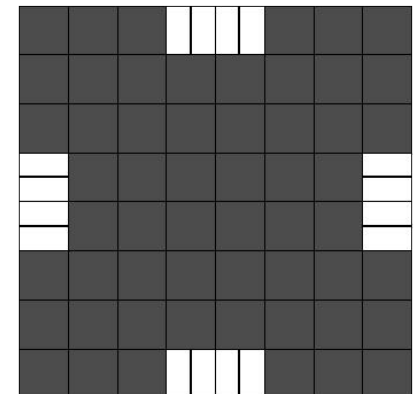
Layer 1



Layer 2




Layer 3

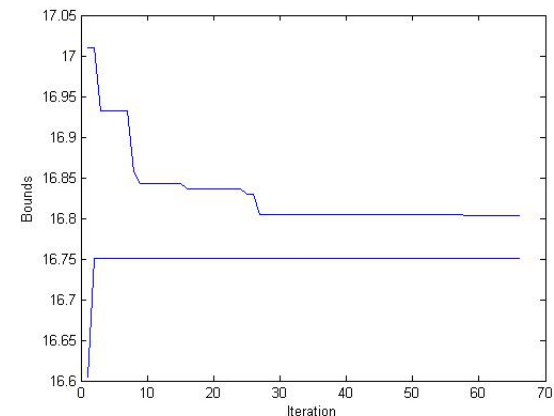
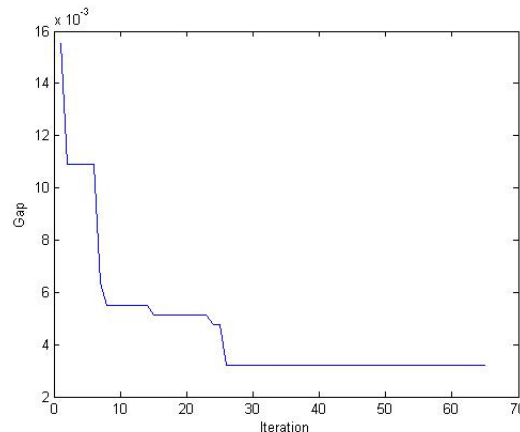


Layer 4

Global Optimization - Outer Approximation

- Approximate the non-linear objective and constraint functions with linear functions.
- Solve a sequence of linear mixed 0-1 problems.
- Guaranteed to converge to global minimizer.

minimize	Compliance		minimize	η
subject to	Compliance constraint		subject to	$c(x^p) + (\nabla c(x^p))^T(x - x^p) - \eta \leq 0, \quad p = 1, \dots, P$
	Weight constraint			$m(x) \leq m_c,$
	Single material selection per layer and element (DMO)			$\sum_{i=1}^{N^i} x_{ijk} = 1, \quad \forall(j, k)$
				$x_{ijk} \in \{0, 1\}, \quad \forall(i, j, k)$
				$\eta \geq 0$



**THANK YOU FOR YOUR
ATTENTION**